Matrices

Assertion & Reason Type Questions

Directions: In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation for Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (A) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false and Reason (R) is true

Q1.

Assertion (A): Matrix $3 \times 3, a_{ij} = \dfrac{i-j}{i+2j}$ cannot be expressed as a sum of symmetric and skew-symmetric matrices.

Reason (R): Matrix $3 \times 3, a_{ij} = \dfrac{i-j}{i+2j}$ is neither symmetric nor skew-symmetric.

Answer : (d) Assertion (A) is false and Reason (R) is true

Q2.

Assertion (A): Scalar matrix $A = [a_{ij}] = \begin{cases} k; & i = j \\ 0; & i \neq j \end{cases}$ where, k is a scalar, is an identity matrix when $k = 1$.

Reason (R): Every identity matrix is not a scalar matrix.

Answer : (c) Assertion (A) is true but Reason (R) is false

Q3.

Assertion (A): $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ **is a diagonal matrix.**

Reason (R): $A = [a_{ij}]$ **is a square matrix such that** $a_{ij} = 0, \forall\, i \neq j$, **then** A **is called diagonal matrix.**

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation for Assertion (A)

Q4.

Assertion (A): $B = \begin{bmatrix} -\frac{1}{2} & \sqrt{5} & 2 & 3 \end{bmatrix}_{1 \times 4}$ **is a row matrix.**

Reason (R): If $B = [b_{ij}]_{1 \times n}$ **is a row matrix, then its order is** $1 \times n$.

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation for Assertion (A)

Q5.

Assertion (A): If $\begin{bmatrix} xy & 4 \\ z+5 & x+y \end{bmatrix} = \begin{bmatrix} 4 & w \\ 0 & 4 \end{bmatrix}$, **then** $x = 2, y = 2, z = -5$ **and** $w = 4$.

Reason (R): Two matrices are equal, if their orders are same and their corresponding elements are equal.

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation for Assertion (A)

Q6. Assertion (A): The product of two diagonal matrices of order 3 × 3 is also a diagonal matrix.

Reason (R): Matrix multiplication is always non-commutative.

Answer : (c) Assertion (A) is true but Reason (R) is false

Q7. Let A be a square matrix of order 3 satisfying AA′ = 1.

Assertion (A): $A' = A^{-1}$.

Reason (R): (AB)′ = B′ A′.

Answer : (b) Both Assertion (A) and Reason (R) are true but Reason (A) is not the correct explanation of Assertion (A)

Q8.

Assertion (A): Let $A = [a_{ij}]$ be an $m \times n$ matrix and O be an $m \times n$ zero matrix, then $A + O = O + A = A$. In other words, O is the additive identity for matrix addition.

Reason (R): Let $A = [a_{ij}]_{m \times n}$ be any matrix, then we have another matrix as $-A = [-a_{ij}]_{m \times n}$ such that $A + (-A) = (-A) + A = O$. Then, $-A$ is the additive inverse of A or negative of A.

Answer : (b) Both Assertion (A) and Reason (R) are true but Reason (A) is not the correct explanation of Assertion (A)

Q9. **Assertion (A):** For multiplication of two matrices A and B, the number of columns in A should be less than the number of rows in B.

Reason (R): For getting the elements of the product matrix, we take rows of A and columns of B, multiply them elementwise and take the sum.

Answer : (d) Assertion (A) is false and Reason (R) is true

Q10.

Assertion (A): If $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then $(A + B)^2 = A^2 + B^2 + 2AB$.

Reason (R): For the matrices A and B given in Assertion (A), $AB = BA$.

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation for Assertion (A)

Q11.

Assertion (A): If $A = \frac{1}{3}\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & -2 & -1 \end{bmatrix}$, **then** $A(A^T) = I$.

Reason (R): For any square matrix A, $(A^T)^T = A$.

Answer : (b) Both Assertion (A) and Reason (R) are true but Reason (A) is not the correct explanation of Assertion (A)

Q12. For any square matrix A with real number entries, consider the following statements:

Assertion (A): A + A' is a symmetric matrix.

Reason (R): A – A' is a skew-symmetric matrix.

Answer : (b) Both Assertion (A) and Reason (R) are true but Reason (A) is not the correct explanation of Assertion (A)

Q13. Assertion (A) A 2×2 matrix $A = [a_{ij}]$, whose elements are given by $a_{ij} = i \times j$, is $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.

Reason (R) If A is a 4×2 matrix, then the elements in A is 5.

Q14. Assertion (A) The matrix $A = \begin{bmatrix} 3 & -1 & 0 \\ 3/2 & 3\sqrt{2} & 1 \\ 4 & 3 & -1 \end{bmatrix}$ is rectangular matrix of order 3.

Reason (R) If $A = [a_{ij}]_{m \times 1}$, then A is column matrix.

Q15 **Assertion** (A) Scalar matrix
$A = [a_{ij}] = \begin{cases} k; & i = j \\ 0; & i \neq j \end{cases}$, where k is a scalar,

is an identity matrix when $k = 1$.

Reason (R) Every identity matrix is not a scalar matrix.

Q16 **Assertion** (A) If $A = \begin{bmatrix} 3 & 1 \\ -5 & x \end{bmatrix}$, then $(-A)$ is given by $\begin{bmatrix} -3 & -1 \\ 5 & -x \end{bmatrix}$.

Reason (R) The negative of a matrix is given by $-A$ and is defined as $-A = (-1)A$.

Assertion (A) If $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$, then $3A - C = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$.

Reason (R) If the matrices A and B are of same order, say $m \times n$, satisfy the commutative law, then $A + B = B + A$.

Assertion (A) If $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, then $A + B = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$.

Reason (R) Two different matrices can be added only if they are of same order.

Assertion (A) If $\begin{bmatrix} xy & 4 \\ z+5 & x+y \end{bmatrix} = \begin{bmatrix} 4 & w \\ 0 & 4 \end{bmatrix}$, then $x = 2$, $y = 2$, $z = -5$ and $w = 4$.

Reason (R) Two matrices are equal, if their orders are same and their corresponding elements are equal.

Assertion (A) If $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$, then AB and BA both are defined.

Reason (R) For the two matrices A and B, the product AB is defined, if number of columns in A is equal to the number of rows in B.

Let A, B and C are three matrices of same order.

Now, consider the following statements

Assertion (A)
If $A = B$, then $AC = BC$.

Reason (R)
If $AC = BC$, then $A = B$.

Assertion (A)
$\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$

Reason (R)

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Assertion (A) If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the value of k such that $A^2 = kA - 2I$, is -1.

Reason (R) If A and B are square matrices of same order, then $(A + B)(A + B)$ is equal to $A^2 + AB + BA + B^2$.

For the matrices $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, consider the following statements

Assertion (A) $(A + B)' = A' - B'$

Reason (R) $(A - B)' = A' - B'$

Let A and B be two symmetric matrices of order 3.

Assertion (A) $A(BA)$ and $(AB)A$ are symmetric matrices.

Reason (R) AB is symmetric matrix, if matrix multiplication of A with B is commutative.

Assertion (A) If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$, then B is the inverse of A.

Reason (R) If A is a square matrix of order m and if there exists another square matrix B of the same order m, such that $AB = BA = I$, then B is called the inverse of A.

Assertion (A) If $A = \begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix}$, then A^{-1} does not exist.

Reason (R) On using elementary column operations $C_2 \to C_2 - 2C_1$ in the following matrix equation

$\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$, we have

$\begin{bmatrix} 1 & -5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$.

13. (c)   14. (d)   15. (c)

Assertion In general, the matrix A of order 2×2 is given by $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$.

Now, $a_{ij} = i \times j, i = 1, 2$ and $j = 1, 2$

$\therefore \quad a_{11} = 1, a_{12} = 2, a_{21} = 2$ and $a_{22} = 4$

Thus, matrix A is $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.

Reason If A is a 4×2 matrix, then A has $4 \times 2 = 8$ elements.

Assertion $A = \begin{bmatrix} 3 & -1 & 0 \\ \frac{3}{2} & 3\sqrt{2} & 1 \\ 4 & 3 & -1 \end{bmatrix}$ is a square matrix of order 3.

Reason In general, $A = [a_{ij}]_{m \times 1}$ is a column matrix.

A scalar matrix $A = [a_{ij}] = \begin{cases} k; & i = j \\ 0; & i \neq j \end{cases}$ is an identity matrix when $k = 1$.

But every identity matrix is clearly a scalar matrix.

We define $-A = (-1)A$.

If $A = \begin{bmatrix} 3 & 1 \\ -5 & x \end{bmatrix}$, then $-A$ is given by

$$-A = (-1)A = (-1)\begin{bmatrix} 3 & 1 \\ -5 & x \end{bmatrix} = \begin{bmatrix} -3 & -1 \\ 5 & -x \end{bmatrix}$$

$$3A - C = 3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6-(-2) & 12-5 \\ 9-3 & 6-4 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$$

The given matrices are $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$.

Then, $A + B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 2+1 & 4+3 \\ 3-2 & 2+5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$$

We have, $\begin{bmatrix} xy & 4 \\ z+5 & x+y \end{bmatrix} = \begin{bmatrix} 4 & w \\ 0 & 4 \end{bmatrix}$

On comparing both the matrices, we get

$$z + 5 = 0 \Rightarrow z = -5$$

$$4 = w \Rightarrow w = 4$$

$$x + y = 4 \text{ and } xy = 4 \Rightarrow y = \frac{4}{x}$$

$\therefore \quad x + \frac{4}{x} = 4 \Rightarrow x^2 + 4 = 4x$

$\Rightarrow \quad x^2 - 4x + 4 = 0$

$\Rightarrow \quad x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(4)}}{2(1)}$

$\Rightarrow \quad x = \dfrac{4 \pm \sqrt{16 - 16}}{2} = \dfrac{4}{2} = 2$

$\therefore \quad y = 4 - x = 4 - 2 = 2$

The given matrices are

$A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$

Order of $A = 2 \times 3$; Order of $B = 3 \times 2$

Since, number of columns in A is equal to the number of rows in B.

$\Rightarrow AB$ is defined.

Also, number of columns in B is equal to the number of rows in A.

$\therefore$ The product BA is also defined.

Assertion $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}=\begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+28 \end{bmatrix}$

$$=\begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}=\begin{bmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{bmatrix}$$

$$=\begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$$

Hence, $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$

Reason Here, $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

$$=\begin{bmatrix} -1+0+6 & 1-2+9 & 0+2+12 \\ 0+0+0 & 0+(-1)+0 & 0+1+0 \\ -1+0+0 & 1-1+0 & 0+1+0 \end{bmatrix}$$

$$=\begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

and $\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

$$=\begin{bmatrix} -1+0+0 & -2+1+0 & -3+0+0 \\ 0+0+1 & 0-1+1 & 0+0+0 \\ 2+0+4 & 4+3+4 & 6+0+0 \end{bmatrix}$$

$$=\begin{bmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$$

Hence, $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

$$\neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Assertion Given, $A^2 = kA - 2I$

$\Rightarrow \quad AA = kA - 2I$

$\Rightarrow \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$

By definition of equality of matrix, the given matrices are equal and their corresponding elements are equal.

Now, comparing the corresponding elements, we get

$3k - 2 = 1 \Rightarrow k = 1$

$\Rightarrow \quad -2k = -2 \Rightarrow k = 1$

$\Rightarrow \quad 4k = 4 \Rightarrow k = 1$

$\Rightarrow \quad -4 = -2k - 2 \Rightarrow k = 1$

Hence, $k = 1$.

Reason We have,

$(A+B)(A+B) = A(A+B) + B(A+B)$

$= A^2 + AB + BA + B^2$

$A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$

$\therefore A + B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{bmatrix}$

Assertion $(A+B)' = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$

Now, $A' - B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

$\therefore \quad (A+B)' \neq A' - B'$

Reason

Now, $A - B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix}$

$(A-B)' = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

and $A' - B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

$\therefore \quad (A-B)' = A' - B'$

Assertion Since, A and B are symmetric matrices.

$\therefore\ A^T = A$ and $B^T = B$.

Now, to check $A(BA)$ is symmetric.

Consider $[A(BA)]^T = (BA)^T \cdot A^T = (A^T B^T) A^T$

$$= (AB)A = A(BA)$$

So, $[A(BA)]^T = A(BA)$

$\Rightarrow A(BA)$ is symmetric.

Similarly, $(AB)A$ is symmetric.

So, Assertion is true.

Reason Now, $(AB)' = B'A'$

$$= BA$$

This will be symmetric, if A and B is commutative i.e. $AB = BA$.

Hence, both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

Let $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ be two matrices.

Then, $AB = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 4-3 & -6+6 \\ 2-2 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Also, $BA = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 4-3 & 6-6 \\ -2+2 & -3+4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Thus, B is the inverse of A.

Assertion We have, $A = IA$

i.e. $$\begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & -1/5 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} 1/10 & 0 \\ 0 & 1 \end{bmatrix} A$$

$\left[\text{applying } R_1 \to \frac{1}{10} R_1\right]$

$$\Rightarrow \begin{bmatrix} 1 & -1/5 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1/10 & 0 \\ 1/2 & 1 \end{bmatrix}$$

[applying $R_2 \to R_2 + 5R_1$]

We have all zeroes in the second row of the left hand side matrix of above equation.

Therefore, A^{-1} does not exist.

Reason The given matrix equation is

$$\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$$

$\because$ The column transformation $C_2 \to C_2 - 2C_1$ is applied.

$\therefore$ This transformation is applied on LHS and on second matrix of RHS.

Thus, we have $\begin{bmatrix} 1 & -5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$.

Assertion (A): If A is a square matrix such that $A^2 = A$, then $(I + A)^2 - 3A = I$

Reason (R): AI = IA = A

Ans. Option (A) is correct.

Explanation: AI = IA = A is true.

Hence R is true.

Given $\quad A^2 = A,$

$$\therefore \quad (I+A)^2 - 3A = I^2 + 2IA + A^2 - 3A$$

$$= I + 2A + A - 3A$$

$$= I$$

Hence A is true.

R is the correct explanation for A.

Assertion (A): $\begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ is a scalar matrix.

Reason (R): If all the elements of the principal diagonal are equal, it is called a scalar matrix.

Ans. Option (C) is correct.

Explanation: In a scalar matrix the diagonal elements are equal and the non-diagonal elements are zero. Hence R is false.

A is true since the diagonal elements are equal and the non-diagonal elements are zero.

Assertion (A): $(A + B)^2 \neq A^2 + 2AB + B^2$.

Reason (R): Generally $AB \neq BA$

Ans. Option (A) is correct.

Explanation: For two matrices A and B, generally $AB \neq BA$.

i.e., matrix multiplication is not commutative.

$\therefore$ R is true

$$(A+B)^2 = (A+B)(A+B)$$

$$= A^2 + AB + BA + B^2$$

$$\neq A^2 + 2AB + B^2$$

$\therefore$ A is true

R is the correct explanation for A.

A and B are two matrices such that both AB and BA are defined.

Assertion (A): $(A + B)(A - B) = A^2 - B^2$

Reason (R): $(A + B)(A - B) = A^2 - AB + BA - B^2$

Ans. Option (D) is correct.

Explanation: For two matrices A and B, even if both AB and BA are defined, generally $AB \neq BA$.

$(A + B)(A - B) = A^2 - AB + BA - B^2$.

Since $AB \neq BA$, $(A + B)(A - B) \neq A^2 - B^2$.

Hence R is true and A is false.

Let A and B be two symmetric matrices of order 3.

Assertion (A): A(BA) and (AB)A are symmetric matrices.

Reason (R): AB is symmetric matrix if matrix multiplication of A with B is commutative.

Ans. Option (B) is correct.

Explanation: Generally $(AB)' = B'A'$
If $AB = BA$, then $(AB)' = (BA)' = A'B' = AB$
Since $(AB)' = AB$, AB is a symmetric matrix. Hence R is true.

$$A(BA) = (AB)A = ABA$$
$$(ABA)' = A'B'A' = ABA.$$

$A(BA)$ and $(AB)A$ are symmetric matrices. Hence A is true.
But R is not the correct explanation for A.

Assertion (A): If the matrix $P = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & 3 \end{bmatrix}$ is a symmetric matrix, then $a = \frac{-2}{3}$ and $b = \frac{3}{2}$.

Reason (R): If P is a symmetric matrix, then $P' = -P$.

Ans. Option (C) is correct.

Explanation: If P is a symmetric matrix, then $P' = P$.
Hence R is false.
As P is a symmetric matrix, $P' = P$

$$\therefore \begin{bmatrix} 0 & 3 & 3a \\ 2b & 1 & 3 \\ -2 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$$

$\therefore$ By equality of matrices, $a = \frac{-2}{3}$ and $b = \frac{3}{2}$.
Hence A is true.

Assertion (A): If A is a symmetric matrix, then $B'AB$ is also symmetric.

Reason (R): $(ABC)' = C'B'A'$

Ans. Option (A) is correct.

Explanation: For three matrices A, B and C, if ABC is defined then $(ABC)' = C'B'A'$.
Hence R is true.
Given that A is symmetric $\Rightarrow A' = A$
$(B'AB)' = B'A'(B')' = B'AB.$
Hence A is true.
R is the correct explanation for A.

Assertion (A): If A and B are symmetric matrices, then $AB - BA$ is a skew symmetric matrix

Reason (R): $(AB)' = B'A'$

Ans. Option (A) is correct.

Explanation: $(AB)' = B'A' \Rightarrow$ R is true.
Given that A and B are symmetric matrices.
$\therefore A' = A$ and $B' = B$
$(AB - BA)' = (AB)' - (BA)'$
$= B'A' - A'B' = BA - AB$
Since $(AB - BA)' = -(AB - BA)$,
$AB - BA$ is skew symmetric.
Hence A is true.
R is the correct explanation for A.

Assertion (A): A matrix $A = [1\ \ 2\ \ 0\ \ 3]$ is a row matrix of order 1×4.

Reason (R): A matrix having one row and any number of column is called a row matrix.

Assertion (A): If $\begin{bmatrix} x^2 - 4x & x^2 \\ x^2 & x^3 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -x+2 & 1 \end{bmatrix}$, then the value of $x = 1$.

Reason (R): Two matrices $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$ of same order $m \times n$ are equal, if $a_{ij} = b_{ij}$ for all $i = 1, 2, 3, \ldots m$ and $j = 1, 2, 3, \ldots n$.

Assertion (A): If A and B are symmetric matrices of same order then $AB - BA$ is also a symmetric matrix.

Reason (R): Any square matrix A is said to be skew-symmetric matrix if $A = -A^T$, where A^T is the transpose of matrix A.

Assertion (A): If A is an invertible square matrix, then A^T is invertible.

Reason (R): Inverse of invertible symmetric matrix is a symmetric matrix.

Assertion (A): If A is an invertible matrix of order 3 and $|A| = 5$ then, $|adj\ A| = 25$.

Reason (R): If B is a non-singular matrix of order n. Then, $|adj\ A| = |A|^{n-1}$.

Assertion (A): If $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ then $|adj\ (adj\ A)| = 16$.

Reason (R): $|\text{adj}\ (\text{adj}\ A)| = |A|^{(n-1)^2}$

Answers